Book: Bifurcation Analysis of Fluid Flows
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(note: also use that $\delta(w)$ is bounded, i.e. $\delta(w) \leq K_\delta \|w\|_{\mathcal{V}}$)
Remark:
this exercise shows that if the right-hand side of the weak form $a(w,u) = F(w)$ is perturbed a little bit, the difference between the respective solutions is bounded, and tends to zero if the perturbation tends to zero, i.e. we have stability.

1. $a(w,u) = F(w)$ and $a(w,\tilde{u}) = F(w) + \delta(w)$. Substract, use linearity of $a(.,.)$, take $w = u - \tilde{u}$, and use linearity of $\delta(.)$ to obtain $a(u-\tilde{u}, u-\tilde{u}) = \delta(\tilde{u}-u)$.

2. now use coercivity of $a(.,.)$ and boundedness of $\delta(.)$ to obtain $c\|u-\tilde{u}\|_{\mathcal{V}}^2 \leq K_\delta \|u-\tilde{u}\|_{\mathcal{V}}$. From this the asked can be proven.